**Chapter 5**

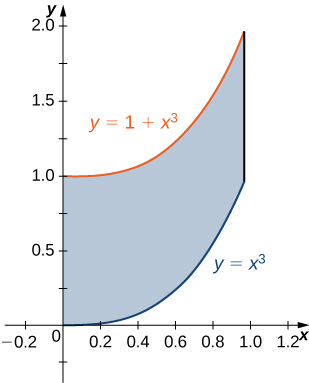
**Multiple Integration**

**5.2 Double Integrals over General Regions**

**Section Exercises**

**In the following exercises, specify whether the region is of Type I or Type II.**

1. The region  bounded by    and  as given in the following figure.



Answer: Type I but not Type II

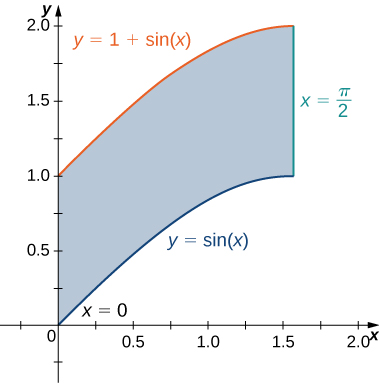
1. Find the average value of the function  on the region graphed in the previous exercise.

Answer: 

1. Find the area of the region given in the previous exercise.

Answer: 

1. The region  bounded by  as given in the following figure.



Answer: Type I but not Type II

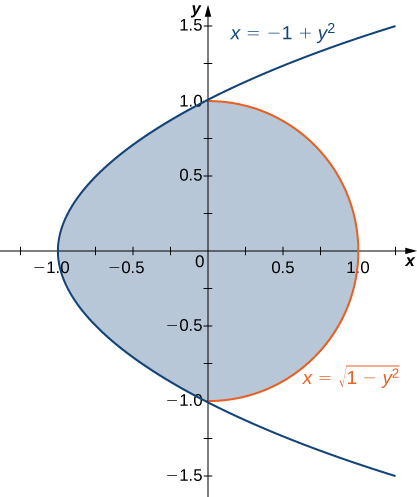
1. Find the average value of the function  on the region graphed in the previous exercise.

Answer: 

1. Find the area of the region  given in the previous exercise.

Answer: 

1. The region  bounded by  and  as given in the following figure.

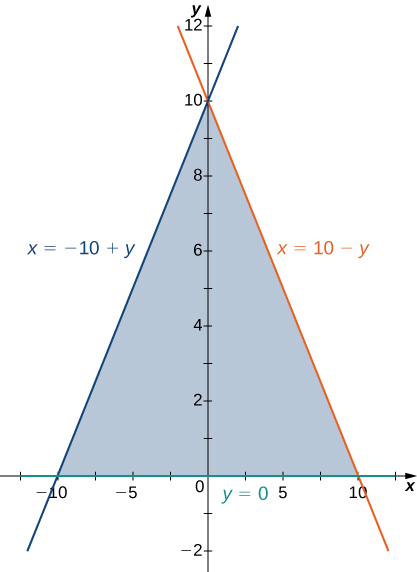


Answer: Type II but not Type I

1. Find the volume of the solid under the graph of the function  and above the region in the figure in the previous exercise.

Answer: 

1. The region  bounded by  as given in the following figure.

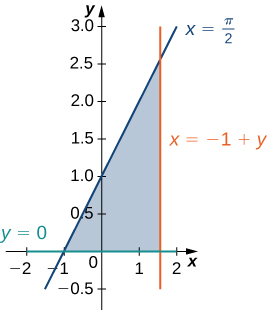


Answer: Type II but not Type I

1. Find the volume of the solid under the graph of the function  and above the region in the figure from the previous exercise.

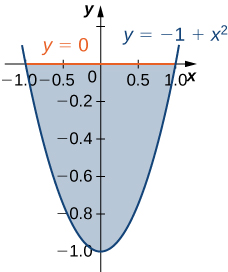
Answer: 

1. The region  bounded by   as given in the following figure.



Answer: Type I and Type II

1. The region  bounded by  and  as given in the following figure.



Answer: Type I and Type II

1. Let  be the region bounded by the curves of equations  and  Explain why  is neither of Type I nor II.

Answer: The region  is not of Type I: it does not lie between two vertical lines and the graphs of two continuous functions  and  The region  is not of Type II: it does not lie between two horizontal lines and the graphs of two continuous functions  and 

1. Let be the region bounded by the curves of equations  and  and the *-*axis. Explain why  is neither of Type I nor II.

Answer: The region  is not of Type I: it does not lie between two vertical lines and the graphs of two continuous functions  and  The region  is not of Type II: it does not lie between two horizontal lines and the graphs of two continuous functions  and 

In the following exercises, evaluate the double integral  over the region 

1.  and 

Answer: 

1.  and 

Answer: 

1.  and 

Answer: 

1.  and 

Answer: 

1.  and  is the triangular region with vertices 

Answer: 

1.  and  is the triangular region with vertices 

Answer: 

**Evaluate the iterated integrals.**

1. 

Answer: 

1. 

Answer: 

1. 

Answer: 

1. 

Answer: 

1. 

Answer: 

1. 

Answer: 

1. Let  be the region bounded by  and the- and-axes.
2. Show that  by dividing the region  into two regions of Type I.
3. Evaluate the integral .

Answer: a. Answers may vary; b. 

1. Let be the region bounded by    and the -axis.
2. Show that  by dividing  into two regions of Type I.
3. Evaluate the integral 

Answer: a. Answers may vary; b. 

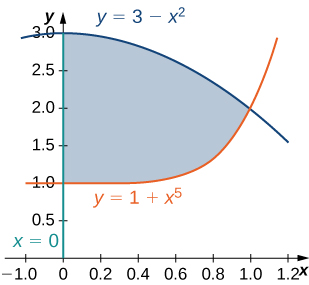
1. Show that  by dividing the region  into two regions of Type I, where 
2. Evaluate the integral 

Answer. a. Answers may vary; b. 

1. Let  be the region bounded by  and 
2. Show that  by dividing the region  into two regions of Type II, where .
3. Evaluate the integral 

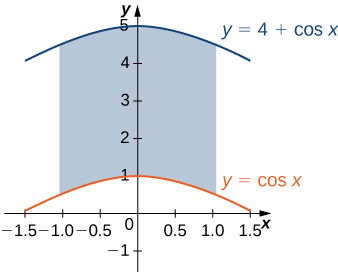
Answer: a. Answers may vary; b. 

1. The region  bounded by  and  is shown in the following figure. Find the area  of the region 



Answer: 

1. The region  bounded by  and  is shown in the following figure. Find the area  of the region 



Answer: 

1. Find the area  of the region 

Answer: 

1. Let  be the regionbounded by  and the -axis. Find the area  of the region 

Answer: 

1. Find the average value of the function  on the triangular region with vertices  and 

Answer: 

1. Find the average value of the function  on the triangular region with vertices  and 

Answer: 

**In the following exercises, change the order of integration and evaluate the integral.**

1. 

Answer: 

1. 

Answer: 

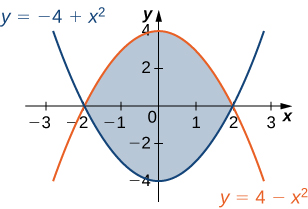
1. 

Answer: 

1. 

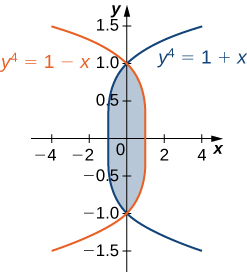
Answer: 

1. The region  is shown in the following figure. Evaluate the double integral  by using the easier order of integration.



Answer: 

1. The region  is given in the following figure. Evaluate the double integral  by using the easier order of integration.



Answer: 

1. Find the volume of the solid under the surface  and above the region bounded by  and 

Answer: 

1. Find the volume of the solid under the plane  and above the region determined by  and 

Answer: 

1. Find the volume of the solid under the plane  and above the region bounded by  and 

Answer: 

1. Find the volume of the solid under the surface  and above the plane region bounded by  and 

Answer: 

1. Let  be a positive, increasing, and differentiable function on the interval  Show that the volume of the solid under the surface  and above the region bounded by    and  is given by 

Answer: This is a proof; therefore, no answer is provided.

1. Let  be a positive, increasing, and differentiable function on the interval  and let  be a positive real number. Show that the volume of the solid under the surface  and above the region bounded by  and  is given by 

Answer: This is a proof; therefore, no answer is provided.

1. Find the volume of the solid situated in the first octant and determined by the planes  

Answer: 

1. Find the volume of the solid situated in the first octant and bounded by the planes  

Answer: 

1. Find the volume of the solid bounded by the planes  and 

Answer: 

1. Find the volume of the solid bounded by the planes  

Answer: 

1. Let  and  be the solids situated in the first octant under the planes  and respectively, and let  be the solid situated between 
2. Find the volume of the solid 
3. Find the volume of the solid 
4. Find the volume of the solid  by subtracting the volumes of the solids 

Answer: a.  b.  c. 

1. Let  be the solids situated in the first octant under the planes  and  respectively, and let  be the solid situated between 
2. Find the volume of the solid 
3. Find the volume of the solid 
4. Find the volume of the solid by subtracting the volumes of the solids 

Answer: a.  b.  c. 

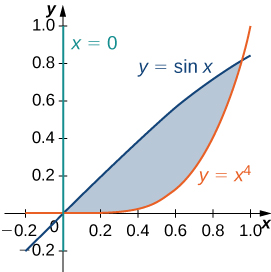
1. Let  be the solids situated in the first octant under the plane  and under the sphere  respectively. If the volume of the solid  is  determine the volume of the solid  situated between  and  by subtracting the volumes of these solids.

Answer: 

1. Let  and be the solids situated in the first octant under the plane  and bounded by the cylinder  respectively.
2. Find the volume of the solid 
3. Find the volume of the solid 
4. Find the volume of the solid  situated between  and  by subtracting the volumes of the solids  and 

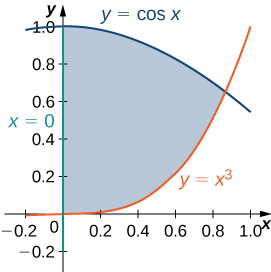
Answer: a.  b.  c. 

1. **[T]** The following figure shows the region  bounded by the curves   and  Use a graphing calculator or CAS to find the -coordinates of the intersection points of the curves and to determine the area of the region Round your answers to six decimal places.



Answer:  

1. **[T]** The region bounded by the curves  is shown in the following figure. Use a graphing calculator or CAS to find the *x*-coordinates of the intersection points of the curves and to determine the area of the region Round your answers to six decimal places.



Answer:  

1. Suppose that  is the outcome of an experiment that must occur in a particular region  in the -plane. In this context, the region  is called the sample space of the experiment and  are random variables. If  is a region included in  then the probability of  being in  is defined as  where  is the joint probability density of the experiment. Here,  is a nonnegative function for which  Assume that a point  is chosen arbitrarily in the square  with the probability density



Find the probability that the point  is inside the unit square and interpret the result.

Answer:  there is a  chance (or  that  is chosen in the unit square

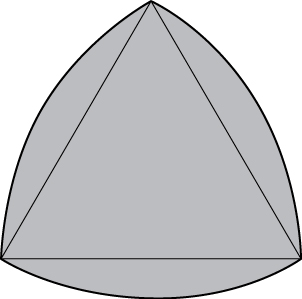
1. Consider  two random variables of probability densities  and  respectively. The random variables  are said to be independent if their joint density function is given by  At a drive-thru restaurant, customers spend, on average,  minutes placing their orders and an additional  minutes paying for and picking up their meals. Assume that placing the order and paying for/picking up the meal are two independent events  and  If the waiting times are modeled by the exponential probability densities

 and 

respectively, the probability that a customer will spend less than 6 minutes in the drive-thru line is given by  where  Find  and interpret the result.

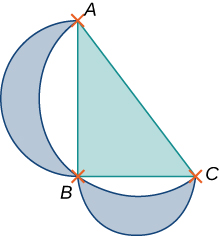
Answer:  there is a  chance that a customer will spend  minutes in the drive-thru line.

1. **[T]** The Reuleaux triangle consists of an equilateral triangle and three regions, each of them bounded by a side of the triangle and an arc of a circle of radius *s* centered at the opposite vertex of the triangle. Show that the area of the Reuleaux triangle in the following figure of side length  is 



Answer: This is a proof; therefore, no answer is provided.

1. **[T]** Show that the area of the lunes of Alhazen, the two blue lunes in the following figure, is the same as the area of the right triangle *ABC*. The outer boundaries of the lunes are semicircles of diameters  respectively, and the inner boundaries are formed by the circumcircle of the triangle 



Answer: This is a proof; therefore, no answer is provided.

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